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Dr. J. C. P. Miller's "Notes on Long-range forecasting"

by C.E.P. Brooks.

Following his week's visit to Stonehouse, Dr. Mil... has sent four "Notes", most of which deal with proposals for shortening the working processes. (Comments are in brackets).

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I. Harmonic Analysis of 24-term sequences. Initial and chain-analysis methods.

For the analysis of data at 25 individual stations, Dr. Miller recommends arithmetical methods instead of the harmonic analysers. A scheme of initial analysis is set out, including correction for non-cyclic change, with various methods of checking. For subsequent analyses a "chain" method is given in which corrections are applied to the harmonic coefficients obtained by the preceding analysis, using only the difference between the value eliminated and that newly introduced. A running check is provided for. If a 24-term analysis of a 72-day interval is used he prefers 3-day sums to 3 daily readings but points out that for higher harmonics corrections for smoothing are considerable; 72-term analysis isalso considered. He calculates that for 25 stations the chain arithmetical analysis would save 4-5 hours every 3 days compared with the Coradi analyser.

(Note: this proposal has not yet been tried out in full, but the chain method was used for a detailed analysis of the 48-day wave at one station. It proved to be very quick but liable to error except in skilled hands. The proportion of man-hours for a forecast which is occupied by the machine analysis is small, so that the question of substituting arithmetical methods is not urgent).

II. On the choice of data and method of analysis etc.

Dr. Miller compares the harmonic coefficients obtained by analysing a series of pressures at Stockholm over a 72-day period, using

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(i) 72-term analysis, daily values, secular constant

$$\Delta' = y_{72} - y_0 = 43.0$$

(ii) " secular constant

$$\Delta = (y_{73}-y_{72}+y_{71}-y_1-y_0-y_{-1})/3$$
$$= 33.47$$

(iii) 24-term analyses, 3-daily values, $\Delta = 33.47$

(iv) " , 3-day means $\Delta = 33.47$

His conclusion is that 24 3-day means make the best basis for analysis.

(i) and (ii) are not practicable for graphical analysis as they involve too many charts. (iv) is preferable to (iii) because the use of 3-day means gives some desirable smoothing; correcting for the reduction of amplitudes may cause trouble in graphical analysis but may not be necessary for harmonics down to the 4th or 5th.

(A brief comparison between (iii) with Δ' and (iv) with Δ was carried out at Stonehouse; the latter showed greater steadiness of coefficients. It may be possible to try (iv) at Dunstable as soon as sufficient 3-day mean charts have been accumulated; meanwhile (iii) is still in use with modified Δ).

The note also discusses the correction for non-cyclic change. This is made up of (a) real change; (b) waves not sub-multiples of 72-days, (c) accidental variations. Only (a) should be introduced into the synthesis but since this is difficult to disentangle it may be better to include the correction in the analysis but not in the synthesis. The discrepancy will not be great in a 12-day forecast. Dr. Miller concludes that the difference between 3-day means is the most convenient measure of non-cyclic change. A least square solution is preferable theoretically but is very cumbersome to apply in practice. (Wadsworth objects that the use of 3-day means would delay the forecasts by a day. The whole question is under consideration at Stonehouse). Finally he discusses

the arguments for and against a fixed time origin, and concludes that the advantages of using substitution or chain-analysis greatly outweigh the small difficulties introduced into phase diagrams and synthesis.

III. Graphical Analysis with a note on Synthesis

Three types of analysis are possible:

(a) 'Initial' analysis, a complete analysis based on all the charts. This is always a necessary start, but unless a fixed origin is retained, it is necessary to make a new complete analysis every 3 days.

(b) 'Substitution' analysis, using a fixed time origin, so that each y_r always has the same multiplier so long as it is used. Only those charts which involve the term y_r need to be replaced by new ones involving $y_r + 24$. Thus many charts may be used again.

(c) 'Chain' analysis as described in note I. We merely calculate terms to add cartographically to $12a_r$, $12b_r$ for one period to convert them to $12a_r$, $12b_r$ for the next period which overlaps by 23 terms. The secular change $\Delta = y_{24} - y_0$ can be automatically included. This is by far the most economical in charts; its most serious drawback is the possibility of transmission and accumulation of errors.

Most of the note deals with the numbers of charts involved and with methods of checking; a large selection of possible checks is given (most of these are rather cumbersome; Brooks and Wadsworth both consider that the harmonic analysis of data for 25 well distributed stations should give an adequate check).

The average numbers of charts required every 3 days are estimated as follows:

(a) Present	(b) Substitution analysis	(c) Chain analysis, initial analysis every		
		36 days	18 days	12 days
127	100	75	88.5	102 without checks.
	138	102	113	124.5 with checks.

Dr. Miller proposes substitution analysis (b) with a subsequent change to chain analysis (c) with a lengthening chain (Wadsworth has not yet accepted the principle of a fixed origin. A trial of chain analysis with a new initial analysis every 18 days is being made at Stonehouse. The results of the first chain of 6 links are moderately good, but not perfect; further experience is obviously required. A corresponding set by substitution analysis is being made for comparison).

An alternative scheme might be tried to include the 48 day wave, i.e. a 48-day (16-term) period for the 48 and 24 day harmonics and a 36 day period for 36, 18, 12 day harmonics. Analysis of 12 daily sheets for 6-day and 3-day harmonics is briefly discussed and it is found that initial analysis is best except that some charts may be used more than once (This part of the procedure has not yet been introduced at Dunstable). Synthesis is considered; since all the harmonics are not used a 'chain' method is not practicable and 'initial' synthesis is necessary each time. (Experience has shown that direct extrapolation of waves does not give satisfactory results and synthesis has accordingly been abandoned. Synthesis using waves calculated over shorter periods is under consideration at Stonehouse)

IV. A discussion on some points connected with the Observational and Mathematical basis of the harmonic analysis method, and on some miscellaneous points.

A number of matters are briefly considered.

(1) Waves are 'quasi-persistent' for $2\frac{1}{2}$-$6\frac{1}{2}$ wave lengths. Consequently a basic period of 72 days may be too long for the shorter waves. The aim being to find the 'frontier' values of the waves, the shortest possible basic periods should be used.

(2) The Harmonic dial can be used in connection with Chain Analysis. (The harmonic dial is already in use at Stonehouse.)

(3) Some experiments are made with an arbitrary series $\chi \sin 45^\circ \chi$, χ increasing linearly from o, as the 3rd harmonic of a 24-term period, and it is found that the secular change of χ introduces other harmonics. In practice this would make forecasting difficult. A trial of analysis of each wave over one wave-length is suggested in spite of the difficulties of 'cross-infection'.

(4) Various analyses of observed pressure are examined, and a warning is given against drawing too many conclusions from overlapping sequences. In general amplitudes diminish with diminishing period, but irregularly and slowly.

(5) The use of symmetry points is discussed and it is considered likely that the correlation method may be more reliable than the use of phases and symmetry slide rules.

(6) The question of harmonic analysis over one or two periods is again discussed and it is considered that there are advantages in using a 72-day basic period but the 48-day wave remains troublesome.

(7) deals briefly with tests of accuracy in forecasting and the drawbacks of using forecast versus actual pressures on individual days at fixed stations are pointed out (Until the 6-day and 3-day waves are introduced in practice, relatively small but rapid changes of pressure cannot be included).

(8) Some minor suggestions are made for facilitating the calculation of symmetry days and indices.

Appendix - Note on Chain and Substitution Analysis

A series of graphical analyses was carried out at Stonehouse by both "chain" and "substitution" methods as set out by Dr. Miller, using 24 3-day mean charts as ordinates. The series have so far been carried to six links, after which a second "initial" analysis was made as a check. The first three harmonics were calculated, and the agreement between the sixth link in both chain and substitution analysis and the check analysis was approximately 80 per cent in each case. Three stations calculated by numerical chain analysis as a further check gave agreement with the a and b coefficients read from the maps to within 1 mb. in all cases. The crests of the waves were usually slightly displaced but the only serious differences occurred on the margins of the charts where the data are uncertain. The differences are probably not greater than the errors inevitable in any method of chart analysis.

The "chain" method requires careful supervision but granted this, we see no reason why it should not prove practicable in application. The comparison is being carried on for another six links.